Nonequatorial Launching to Equatorial Orbits and General Nonplanar Launching

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The general problem of launching out of the plane of an orbit is investigated by considering first the problem of launching from an arbitrary point on earth to a given equatorial orbit. A simplified analysis using a spherical, nonrotating earth model with no atmosphere is discussed. Generalized curves showing the interrelationship of ideal velocity to burnout conditions and the true heading to burnout conditions are presented for several values of launch latitude and several orbit altitudes. From these, a curve for each orbit altitude showing the minimum ideal velocity as a function of launch latitude is derived. Curves for time and angular range of the ascent trajectory for several orbit altitudes also are included. A method is given for using these curves when the desired circular orbit is inclined.

Nomenclature

a = semimajor axis
c = exhaust velocity
e = eccentricity of ascent trajectory
E = eccentric anomaly
E_1 = eccentric anomaly corresponding to θ_1
E_2 = eccentric anomaly corresponding to θ_2
h = orbit altitude
h_{bo} = altitude at burnout
H = heading
H_t = true heading
H_{t_c} = true heading corrected for earth's rotation
i_o = inclination of the plane of the desired orbit
L = longitude difference between the longitude of the launch site and the longitude of intersection of the ascent trajectory with the equator
L_o = longitude difference between the longitude of the launch site and the longitude of the line of nodes of plane of desired orbit
m_0 = initial mass of rocket
m_f = final mass of rocket
r = radial distance from center of earth
R = mean radius of earth = 3437.83 naut miles
t = time during ascent to transfer point from launch site
t_1 = time from perigee of ascent orbit to launch site
t_2 = time from perigee of ascent orbit to transfer point
V = velocity
V_a = arrival velocity at altitude h on ascent trajectory
V_{ad} = adjustment velocity
V_{bo} = burnout velocity
V_{bo}'' = reference burnout velocity
V_{boc} = burnout velocity corrected for earth's rotation
V_d = velocity due to drag
V_e = escape velocity from earth's surface
V_{e_h} = escape velocity from altitude h
V_{ea} = adjustment velocity necessary to attain an elliptical orbit
V_{eo} = elliptical orbit velocity at altitude h
V_g = velocity for gravity correction
V_i = ideal velocity
V_{ic} = ideal velocity corrected for earth's rotation
V_o = circular orbit velocity
V_r = velocity due to earth's rotation at the launch site
γ = gravity mass constant for earth = GM = 1.4076×10^{16} ft^3/sec^2
γ = inclination of ascent trajectory plane with equatorial plane
ϕ_{bo} = angle at burnout from local horizontal
ϕ_{bo}'' = reference burnout angle
ϕ_a = angle of V_a from local horizontal
ϕ_{eo} = flight path angle in elliptical orbit at altitude h
θ_1 = true anomaly measured from perigee of ascent trajectory to launch site
θ_2 = true anomaly measured from perigee of ascent trajectory to transfer point
θ = intersection range—the angular range from the launch site to the transfer point
θ'' = reference intersection range
θ_{bo} = angular range from launch site to burnout
θ_{bo}'' = angular range from reference launch site to burnout
τ = latitude of launch site
τ'' = reference launch site when burnout is not at the earth's surface
τ_a = apparent latitude of launch site
ω = angular rotation of earth
transfer point = point on ascent trajectory which has an altitude equal to the desired orbit altitude
up-leg = that part of the ascent orbit where the altitude is increasing
down-leg = that part of the ascent orbit where the altitude is decreasing (those quantities on the down-leg of the ascent trajectory will be primed)

Introduction

It is not always expedient or possible to launch a space vehicle into orbit from a launch site that is located in the plane of the desired orbit. Hence, it becomes important to know how the propulsion requirements are affected by launching from a more convenient site. This, then, will help determine whether it is more worthwhile to pay a penalty in vehicle performance or to change the launch site.

In considering this problem, it became apparent that it is related to the minimum fuel consumption or "Synergy" problem of Oberth,[1] which he expressed by the relation

$$dA/dm = vc \cos\alpha \tag{1}$$

where dA is the increase in energy of the rocket and the other symbols have the usual meaning.

In principle, it is possible to solve this equation for optimum conditions by the calculus of variations. However, the form of the rocket equations leads to expressions that are not integratable. This also is found for an expression based on maximization of the velocity.

A search of the literature disclosed several analyses[2-4] of this and related problems. These invariably assumed that

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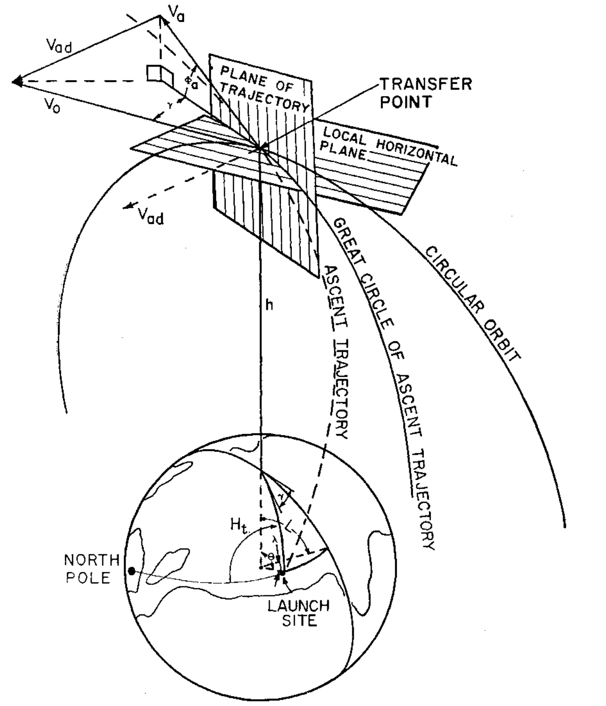

Fig. 1 Pictorial representation of orbit attainment

the desired orbit was joined at apogee of the ascent trajectory. Furthermore, the limits imposed in some analyses[2, 3] lead to trivial solutions.

Consideration of Eq. (1) shows that it is not certain that apogee joining yields the optimum conditions, since, in 5eneral, the factor to be optimized is the amount of fuel used in the correction to orbit velocity. This was checked by an approximate analysis, which showed that apogee joining gave optimum conditions only for in-plane launching.

In view of these factors, it was determined that the problem would need to be solved by numerical integration. In consideration of the potential usefulness to other problems that do not necessarily allow operation at the optimum point, such as rendezvous, the numerical calculation was extended to include a wide range of initial conditions.

Nonequatorial Launch to Equatorial Orbit[5–6]

For simplicity, the calculations are made for a spherical nonrotating earth with no atmosphere and for an inverse square force field. The standard equations of trajectory mechanics are used. The rocket motor burning is assumed to be instantaneous. A pictorial representation of the problem is shown in Fig. 1.

It is assumed that the desired orbit is circular. This assumption is justified by the fact that many satellite applications are satisfied best by orbits that are circular or very nearly so. Also, it is assumed that the desired orbit always lies in the equatorial plane. This permits designation of the launch point by its latitude only, since its longitude has no effect on the problem. The velocity of a circular orbit is

$$V_o^2 = \gamma/(Re + h) \tag{2}$$

The attainment of the desired orbit altitude is achieved by the ascent trajectory. The ascent trajectory is defined as that phase of flight from burnout to the point on the trajectory at which the vehicle attains the desired orbit altitude h. This point will be referred to as the transfer point. During this period, the only influence exerted on the vehicle is the force of gravity. These trajectories are segments of planar elliptical Keplerian paths. The ascent trajectory is depicted in Fig. 2.

The minimum energy at launch for which the ascent trajectory will reach the desired orbit altitude for at least one point in its path is determined by the following relation:

$$a = \frac{(Re\cos\phi_{bo})^2 - (Re+h)^2}{2Re\cos^2\phi_{bo} - 2(Re+h)} \tag{3}$$

Hence any other set of burnout conditions must satisfy the inequality

$$a > \frac{(Re\cos\phi_{bo})^2 - (Re+h)^2}{2Re\cos^2\phi_{bo} - 2(Re+h)} \tag{3a}$$

The maximum energy at launch is assumed to be determined by the escape velocity:

$$V_e^2 = 2\gamma/Re \tag{4}$$

which limits ascent to elliptical orbit segments. Now given the burnout conditions V_{bo}, ϕ_{bo}, $h_{bo} = 0$ (which occur at the launch site for this problem), the ascent trajectory is defined completely. The trajectory elements then are computed from

$$a = \gamma/(V_e^2 - V_{bo}^2) \tag{5}$$

$$e = \left[1 - \frac{Re^2(V_{bo}\cos\phi_{bo})^2}{a\gamma}\right]^{1/2} \tag{6}$$

$$\cos\theta_1 = \frac{a(1-e^2)}{eRe} - \frac{1}{e} \tag{7}$$

$$\cos\theta_2 = \frac{a(1-e^2)}{e(Re+h)} - \frac{1}{e} \tag{8}$$

$$\theta = \theta_2 - \theta_1 \tag{9}$$

$$\tan\frac{E}{2} = \left(\frac{1-e}{1+e}\right)^{1/2}\tan\frac{\theta}{2} \tag{10}$$

$$t = t_2 - t_1 = (a^3/\gamma)\,[E_2 - E_1 - e(\sin E_2 - \sin E_1)] \tag{11}$$

The characteristics at the transfer point are

$$V_a^2 = \gamma\left[\frac{2}{Re+h} - \frac{1}{a}\right] \tag{12}$$

$$\cos\phi_a = \left(\frac{Re}{Re+h}\right)\left(\frac{V_{bo}}{V_a}\right)\cos\phi_{bo} \tag{13}$$

The previous equations completely define the essential quantities of the ascent trajectory when the transfer point is chosen at altitude h on the up-leg of the trajectory. The up-leg of the ascent trajectory is defined as that part of the

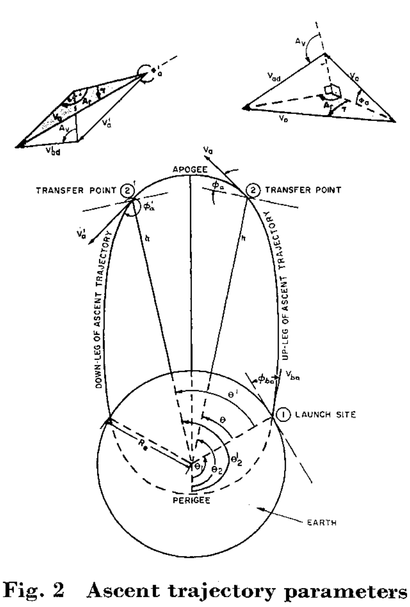

Fig. 2 Ascent trajectory parameters

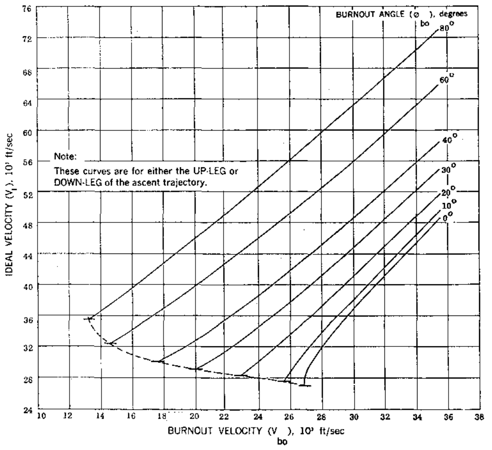

Fig. 3 Ideal velocity requirement for 500-naut-mile equatorial circular orbit, launch site at 0° latitude

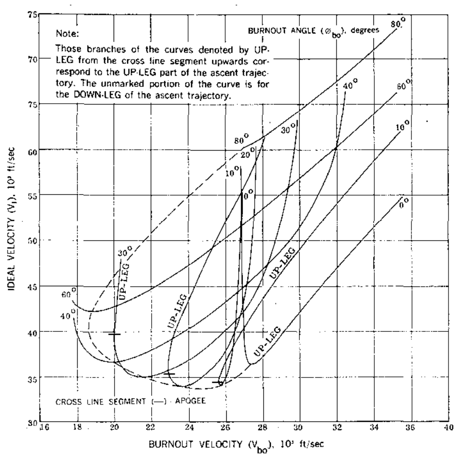

Fig. 4 Ideal velocity requirement for 500-naut-mile equatorial circular orbit, launch site at 20° latitude

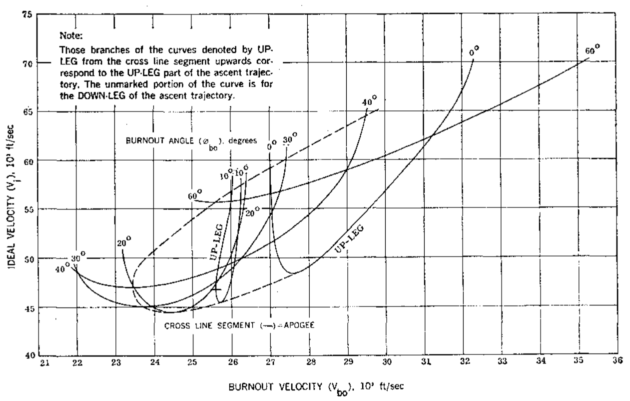

Fig. 5 Ideal velocity requirement for 500-naut-mile equatorial circular orbit, launch site at 50° latitude

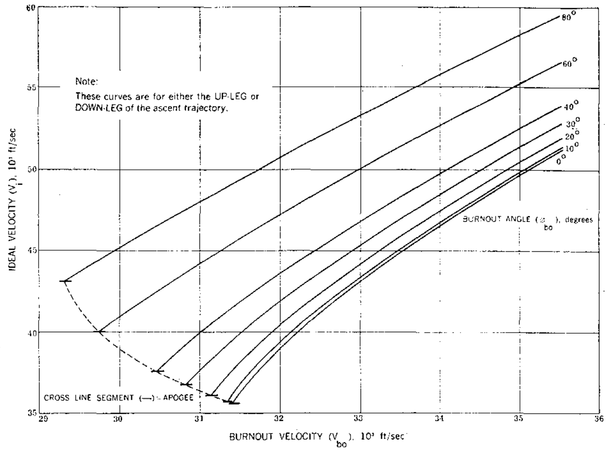

Fig. 6 Ideal velocity requirement for 6000-naut-mile equatorial circular orbit, launch site at 0° latitude

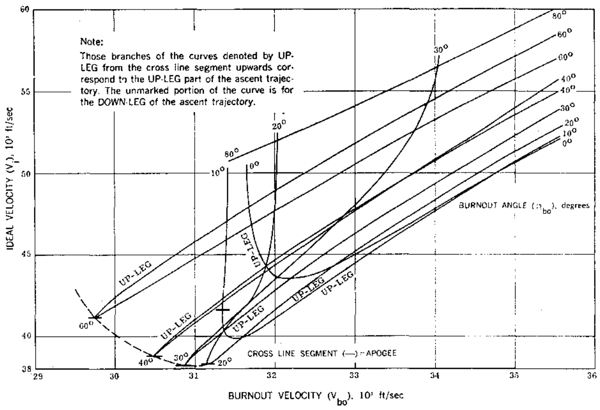

Fig. 7 Ideal velocity requirement for 6000-naut-mile equatorial circular orbit, launch site at 20° latitude

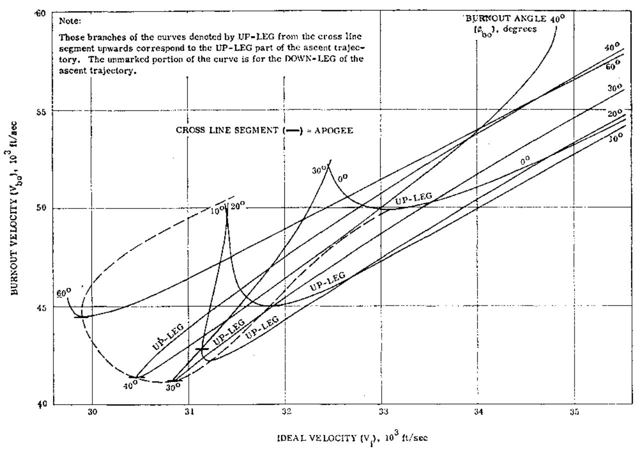

Fig. 8 Ideal velocity requirement for 6000-naut-mile equatorial circular orbit, launch site at 40° latitude

path where the altitude is increasing. Figure 2 shows that for the same burnout conditions the transfer point also can be chosen on the down-leg of the trajectory. When the transfer point is chosen on the down-leg, the related quantities will be primed. From the symmetry of the problem, the previous quantities still are applicable with a few minor revisions. They are

$$\phi_a' = 360° - \phi_a \tag{14}$$

$$\theta_2' = 360° - \theta_2 \tag{15}$$

$$\theta' = \theta_2' - \theta_1 \tag{16}$$

$$t' = t_2' - t_1 = (a^3/\gamma)^{1/2}[E_2' - E_1 - e(\sin E_2' - \sin E_1)] \tag{17}$$

The position of the plane of the ascent trajectory is determined by the launch site latitude τ, the ascent trajectory intersection range θ, and the inclination i_o of the circular orbit. For this analysis, the orbital plane was chosen to be in the equatorial plane where the orbital inclination is 0°. The nonequatorial case is discussed later.

The position of the ascent trajectory with respect to earth is described by the true heading H_t of the trajectory plane; with respect to the orbit plane, it is described by the inclination γ. From Fig. 1 and the use of spherical trigonometry, the inclination γ is

$$\sin\gamma = \sin\tau/\sin\theta \tag{18}$$

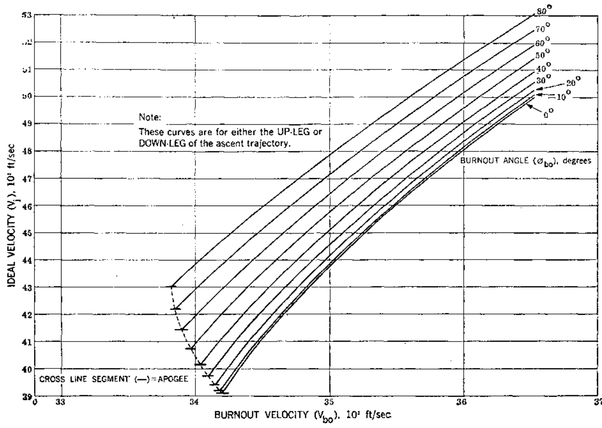

Fig. 9 Ideal velocity requirement for 19,316-naut-mile equatorial circular orbit, launch site at 0° latitude

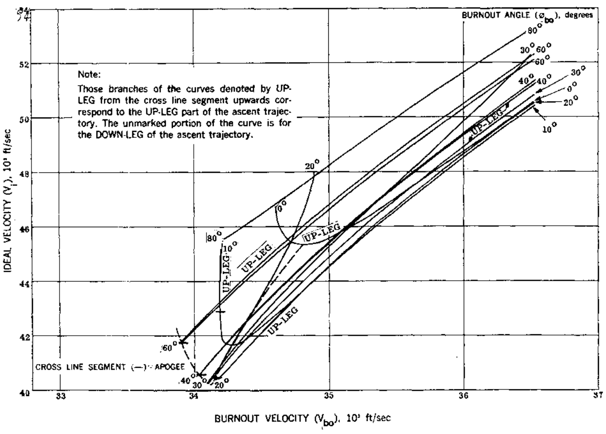

Fig. 10 Ideal velocity requirement for 19,316-naut-mile equatorial circular orbit, launch site at 30° latitude

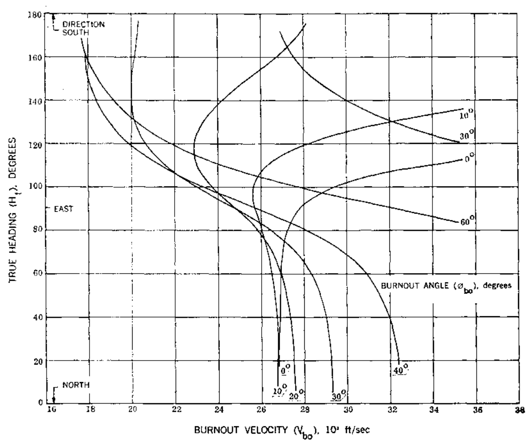

Fig. 11 True heading of burnout velocity vector for 500-naut-mile equatorial circular orbit, launch site at 20° latitude

This equation places a geometric constraint on the possible ascent trajectories requiring that the transfer point on the ascent trajectory lies in the plane of the desired orbit. Therefore, the boundaries on the intersection range θ for a given launch site must be

$$\tau \leq \theta \leq 180 - \tau$$

which in turn places restrictions on the burnout conditions for the ascent trajectories. This constraint demonstrates that, as the latitude of the launch site increases, the possible choices of an ascent trajectory decrease until at the poles there would be at most one ascent trajectory for a given burnout angle and orbital altitude. In other words, if the launch site were chosen at the North (South) Pole, then only one of the burnout conditions need be specified—either the burnout angle or the burnout velocity. The other condition is defined implicitly in the intersection range, which for this case must be 90°.

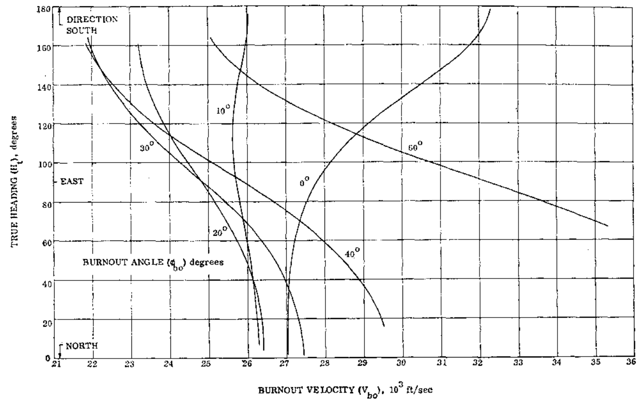

Fig. 12 True heading of burnout velocity vector for 500-naut-mile equatorial circular orbit, launch site at 50° latitude

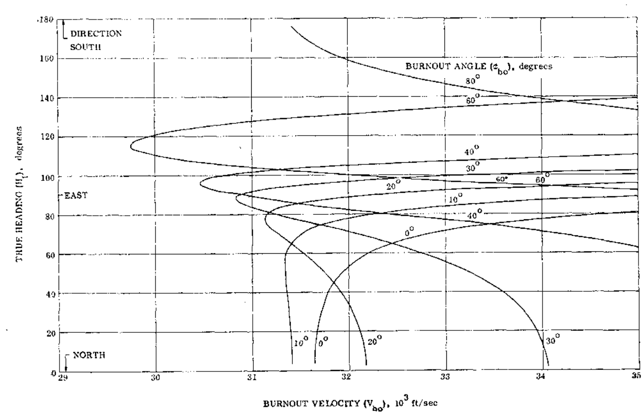

Fig. 13 True heading of burnout velocity vector for 6000-naut-mile equatorial circular orbit, launch site at 20° latitude

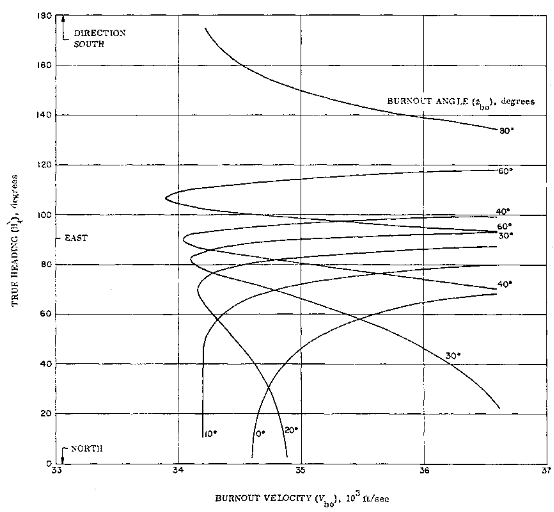

Fig. 14 True heading of burnout velocity vector for 19,316-naut-mile equatorial circular orbit, launch site at 20° latitude

The longitude difference L, the minimum difference between the longitude of the launch site and the longitude of the line of nodes, is

$$\cos L = \cos\theta / \cos\tau \tag{19}$$

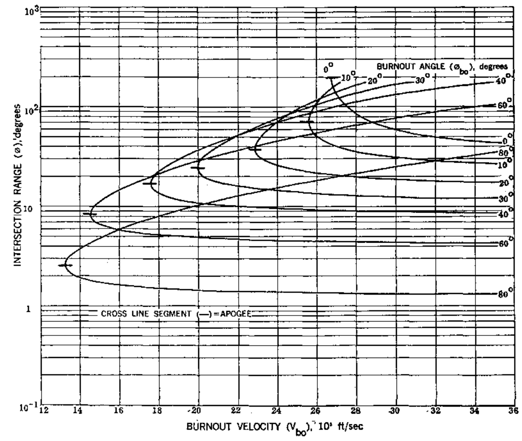

Fig. 15 Ascent trajectory angular range from launch site to transfer point, altitude 500 naut miles

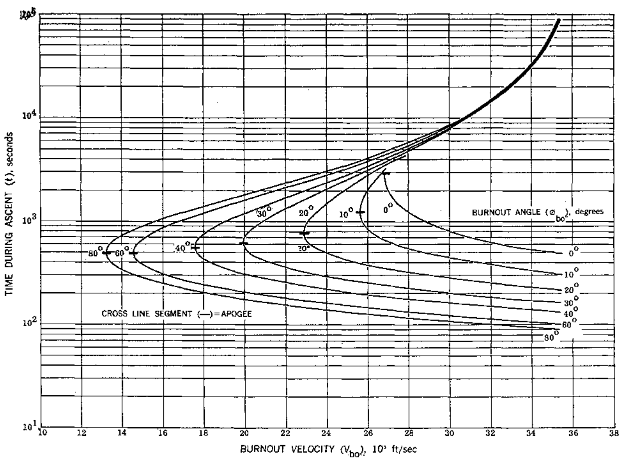

Fig. 16 Ascent trajectory duration from the launch site to the transfer point, altitude 500 naut miles

By proper choice of a launch site or trajectory, a longitude position for the trajectory plane would intersect the orbit so that a desired (or required) epoch for the body in the circular orbit would result.

The heading of the trajectory plane is

$$\cos H = \cos L \sin\gamma \tag{20}$$

From this, the true heading is

$$H_t = 180 - H$$

The two angles of most interest are the inclination γ and the true heading H_t. The inclination of the plane of the ascent trajectory is of importance because it determines how much the velocity vector at the transfer point must be rotated in order to be in the plane of the orbit. The true heading H_t is important at the launch site because it determines the direction in which the vehicle will be fired. The inclination and the true heading are related by

$$\cos\gamma = \cos\tau \sin H_t \tag{21}$$

The space vehicle arrives at the transfer point via the ascent trajectory with a velocity of magnitude V_a which makes an angle ϕ_a with the local horizontal. At this point, a velocity must be added of such magnitude and direction that the resultant velocity will be that of the desired orbit. This additional velocity is called the adjustment velocity V_{ad} (sometimes miscalled the injection velocity) and is shown in Fig. 1. From Fig. 2 and plane trigonometry, the adjustment velocity can be found from

$$V_{ad}^2 = V_o^2 + V_a^2 - 2V_oV_a \cos\phi_a \cos\gamma \tag{22}$$

In a practical situation where rocket burning time is finite, a correction must be added for the travel during the burning period. This is not considered in this paper.

The ideal velocity to attain an equatorial circular orbit is

$$V_i = |\mathbf{V}_{bo}| + |\mathbf{V}_{ad}| \tag{23}$$

Using the basic rocket equation,

$$V = c \ln(m_o/m_f) \tag{24}$$

the ideal velocity can be translated into propellant and payload weights.

For the practical situation, there are other velocities that must be considered. The space vehicle has an additional velocity V_r due to the rotation of the earth. Also, the velocity is affected by drag in the earth's atmosphere and by the gravity during its burning periods. The drag correction velocity will be designated as V_d and the gravity correction velocity as V_g. The ideal velocity can be corrected easily for these effects by

$$V_i = |\mathbf{V}_{bo} - \mathbf{V}_r| + |\mathbf{V}_{ad}| + |\mathbf{V}_g| + |\mathbf{V}_d| \tag{25}$$

Discussion of Results

The results of the calculations based on the previous discussion are shown in Figs. 3–20. The ideal velocity curves are shown in Figs. 3–10. These curves represent the propulsion requirements necessary to attain a circular orbit of a specified altitude from a specified launch site via an arbitrary ascent trajectory as described by the conditions at the launch site. The dotted curve represents the minimum ideal velocity possible as a function of flight path angle only. As the latitude of the launch site increases, the possible choices of an ascent trajectory decrease until at the North (South) Pole there would be at most one ascent trajectory possible for one burnout angle. It should be noted that, although there are two different ideal velocities possible for the same ascent trajectory, this does not imply that the solution is not unique. It only means that one point is on the up-leg of the ascent trajectory, and the other is on the down-leg. Those segments of the constant burnout angle curves have been designated as to whether they represent either the up-leg or the down-leg. That point also has been marked (if it exists) on the curve of constant burnout angle, which represents the ascent trajectory that reaches the orbit altitude at the apogee of its trajectory. For a given burnout angle, only one such trajectory is possible. It is interesting to note that the ascent trajectory that gives the minimum ideal velocity for a particular burnout angle is not necessarily the one where the transfer point is at the apogee.

A set of graphs related to the ideal velocity curves are the true heading curves. They are Figs. 11–14. These

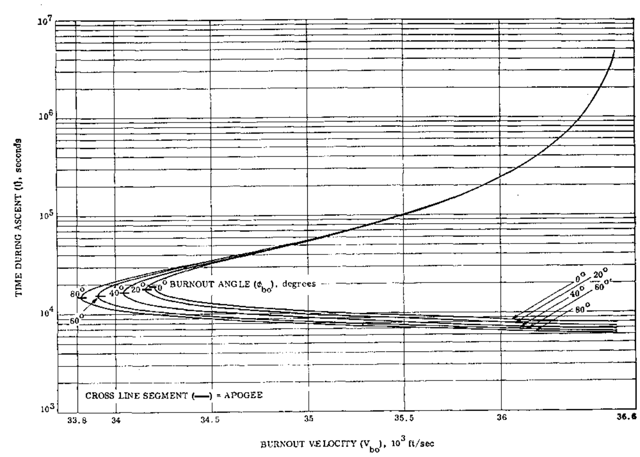

Fig. 17 Ascent trajectory duration from the launch site to the transfer point, altitude 6000 naut miles

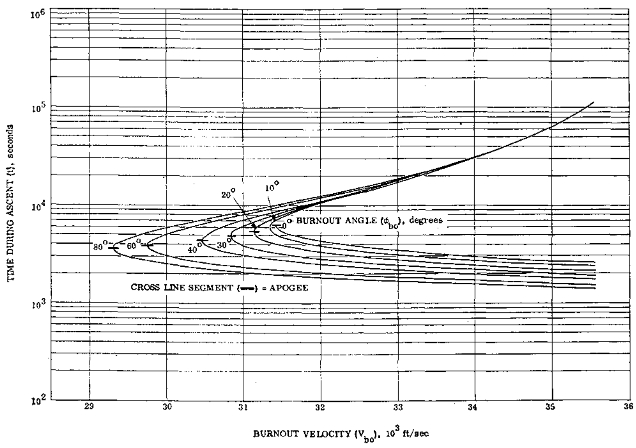

Fig. 18 Ascent trajectory duration from the launch site to the transfer point, altitude 19,316 naut miles

curves represent the direction in which the vehicle is to be fired at the launch site. The north, east, and south directions when firing occurs in the northern hemisphere are marked on the graphs. The segments of the curves representing the up-leg and the down-leg are not designated because it is possible to tell which is which just by looking at the graphs. The up-leg is always the upper half of the curves (if it exists), and the apogee point (if it exists) occurs at the side peak as the curve turns on itself. If there is any doubt where the apogee point is, refer to the proper ideal velocity curve. The range and time curves for the ascent trajectory comprise Figs. 15–18. These curves provide the angular range and the time from the launch site to the transfer point at the altitude of the desired orbit. Since these quantities are characteristics of the ascent trajectory only, it is not necessary to relate them to the launch site. They are to be used concurrently with any of the ideal velocity or true heading curves. The minimum ideal velocity curve is shown in Fig. 20, and the corresponding burnout conditions for one altitude are shown in Fig. 19. These curves show how the minimum ideal velocity varies with the launch site and defines by its burnout conditions the corresponding ascent trajectory that will give this minimum velocity. These curves were obtained by reading the minimum point on the dotted curve from the set of ideal velocity curves. The minimum ideal velocity curves point out the fact that the relation between the launch site latitude and the minimum ideal velocity is almost linear. These curves are helpful in estimating the penalty for launching out the plane of the orbit.

General Nonplanar Launching

The more general problem of launching to an orbit where the orbital plane is inclined to the equator is solved quickly by considering only the apparent latitude τ_a of the launch site. The apparent latitude is defined as that latitude measured in a rotated geocentric coordinate system where the reference plane is now the plane of the desired orbit. Once the apparent latitude is known, this general problem then becomes identical with the one considered previously. The quantities related to the geographic system of coordinates such as true heading and the inclination of the ascent trajectory plane also must be transformed.

The plane of the desired orbit is defined by its inclination i_o with respect to the equator and the position of the line of nodes. Given the latitude and longitude of the launch site, the inclination of the orbital plane, and longitude of the line of nodes, the apparent latitude can be found from

$$\sin\tau_a = \frac{[1 - (\cos\tau \cos L_o)^2]^{1/2}}{\tan^2\tau + \sin^2 L_o} (\sin^2 L_o \sin i_o + \tan^2\tau \cos i_o) \quad (26)$$

where L_o = |longitude of launch site − longitude of the line of nodes|.

Conclusions

It has been shown in the literature that the minimum ideal velocity for a launch site in the plane of the orbit occurs for a Hohmann-type orbit, with apogee at orbit altitude, as the foregoing results bear out. However, for nonplanar launching, the apogee transfer does not give the minimum possible energy. The minimum point may occur on the up-leg or the down-leg, depending on the combination of height and latitude used. (This should be compared with previous studies, in which apogee transfer was assumed.)

The increased energy required for nonplanar launching is very large for low-altitude orbits and is appreciable at synchronized orbit altitude.

The penalty is a function of individual design and must be calculated for each rocket type. For three-stage rockets typical of 1960 designs, and for an equatorial orbit, launching from Cape Canaveral reduces the payload capacity to 30 to 50% of the values for an equatorial launch site. In some applications, this represents a severe penalty.

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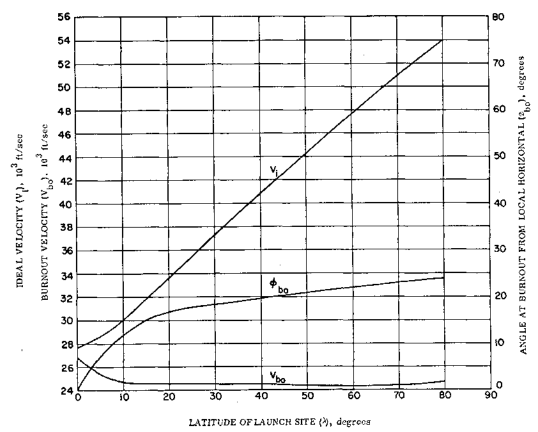
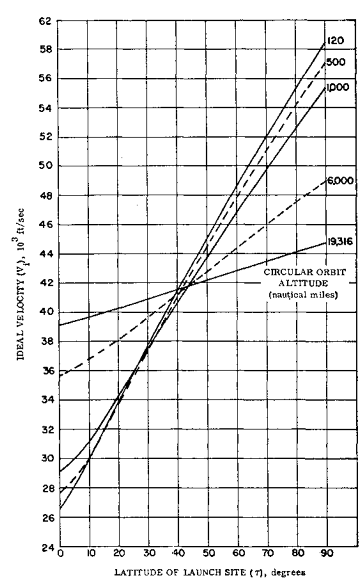


Fig. 19 Minimum ideal velocity and burnout requirement to attain a circular orbit at altitude 500 naut miles when launching out of the plane of the orbit

Fig. 20 Minimum ideal velocity to attain a circular orbit when launching out of the plane of the orbit



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Design Analysis of Earth-Lunar Trajectories: Launch and Transfer Characteristics

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In the design of feasible Earth-lunar trajectories, the launch or boost phase must be linked realistically to the ballistic phase within certain well-defined constraints. This paper presents the relationships between the pertinent parameters from launch to arrival in graphical form which not only are useful in the design of lunar trajectories but also are effective aids in the visualization of the problem's fundamentals. The analysis covers all possible ballistic trajectories to the moon, including lofted trajectories, in as simple a form as is believed possible. The method also is applicable to return trajectories by a simple time reversal technique such that predetermined Earth arrival conditions are assured.

Nomenclature

a = semimajor axis of transfer orbit
A_L = launch azimuth angle (clockwise from local north)
e = eccentricity of transfer orbit
i = inclination of transfer orbital plane to equator
L = unit vector toward launch site at time of launch
r_1 = radius vector at injection, 4078.2860 statute miles (100-naut-mile alt)
r_2 = radius vector at arrival (mean moon distance of 60.27 Earth radii)
S = unit vector toward moon at time of arrival
t_L = hours from midnight to launch time of launch day
T = total flight time (launch to arrival)
T_b = ballistic flight time (injection to arrival)
T_p = powered plus parking orbit flight time (launch to injection)
v_1 = true anomaly at injection
v_2 = true anomaly at arrival
V = injection velocity
V_{cir} = circular velocity at 100-naut-mile alt (25,568 fps)
W = unit vector normal to plane of transfer orbit
α_L = right ascension of L
α_S = right ascension of S
γ = angle of elevation at injection
δ_L = declination of L
δ_S = declination of S
θ_L = longitude of launch site (positive eastward from Greenwich)
μ_e = gravitational constant of Earth, 9.5629993×10^4 statute miles^3/sec^2
φ = total central angle (launch to arrival)
φ_b = ballistic central angle (injection to arrival)
φ_p = powered plus parking orbit central angle (launch to injection)
ω = angular velocity of Earth's rotation (15.04107 deg/hr)
ω_{100} = angular velocity in 100-naut-mile circular orbit (244.91 deg/hr)
GHA = Greenwich hour angle of vernal equinox at 0^h UT of launch day
(All times are given as Universal Time (UT) unless otherwise noted.)

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1. Introduction

In the design of actual hardware for lunar flights, realistic trajectories must be considered for detailed analysis. Although general information about lunar trajectories such as is given in Refs. 1–4 is helpful, the problem of selecting realistic trajectories involves the integration of many widely differing constraints, originating from various sources, which actually may dictate the class of trajectories which can be considered for a particular mission. Many of these constraints are associated with the geometry and dynamics at launch. Therefore, this paper is concerned with those parameters that define launch and ballistic transfer into the vicinity of the moon.

Following the method of Clarke,[5] the problem is divided into two main parts: the geometric constraints that control the plane of the transfer trajectory, and the dynamic constraints that determine the motion in that plane. Although the mathematics is straightforward, the geometry is sufficiently complex to make desirable the aids in visualization presented herein such that the physical significance of the various parameters is made as clear as possible.

2. Geometric Constraints

Consider two unit vectors with origin at the Earth's center: a launch vector **L** in the direction of the launch site at the time of launch, and an outward radial vector **S** in the direction of the moon at the time of arrival. These two vectors define a fixed plane in space which is the plane of the transfer trajectory. This plane forms an angle with the local north at the launch site and time which is the launch azimuth angle A_L. The angle between these vectors is the total central angle φ.